OPTIMAL LOCATION OF ENERGY STORAGE SYSTEMS WITH ROBUST OPTIMIZATION

Nayeem CHOWDHURY
University of Cagliari - Italy
nayeem.chowhdury@enel.com

Giuditta PISANO
University of Cagliari - Italy
giuditta.pisano@diee.unica.it

Fabrizio PILO
University of Cagliari - Italy
pilo@diee.unica.it

Matteo TRONCIA
University of Cagliari - Italy
matteo.troncia@diee.unica.it

ABSTRACT

The integration of intermittent sources of energy and responsive loads in distribution system make the traditional deterministic optimization-based optimal power flow no longer suitable for finding the optimal control strategy for the power system operation. This paper presents a tool for energy storage planning in the distribution network based on AC OPF algorithm that uses a convex relaxation for the power flow equations to guarantee exact and optimal solutions with high algorithmic performances and exploits robust optimization approach to deal with the uncertainties related to renewables and demand. The proposed methodology is applied for storage planning on a distribution network that is representative of a class of networks.

INTRODUCTION

Energy Storage Systems (ESSs) are crucial devices for the upcoming Smart Distribution Systems thanks to the flexibility they introduce in the network operation. It has been seen a rapid improvement in ESS technology, but not yet sufficient to drastically reduce the high investments associated. Thus, optimal planning and management of these devices are essential to identify specific configurations that can justify ESSs installation. This consideration has motivated a strong interest of the researchers in this field that, however, proposed optimization algorithms very seldom capable to deal with uncertainties related, for instance, to the real amount and position of dispersed generation that is going to be connected to the system, the mix of renewable energy sources (RES), the cost of ESS or the level of participation and the cost for activate active demand. Such uncertainties cannot be dealt with stochastic optimization since it is very hard or impossible to define a density function for them, but Robust Optimization (RO) can be applied when the behavior of uncertainties is completely unknown. RO considers a set of uncertainty scenarios instead of a probability distribution over the possible input instances. Each scenario corresponds to one particular realization of the input. The constraints in the robust optimization should be considered as hard constraints which means they have to be maintained for all the instances in the uncertainty set. In RO, the modeler aims at finding decisions that are optimal for the worst-case realization of the uncertainties within a given set.

In the literature, various mathematical models have been proposed to address the optimal placement of ESSs in power systems (see for instance [1]-[6]). The problem of identifying the optimal location of ESS in the electric grid is highly dimensional and non-convex. Since the placement of ESS requires analysis of the impact of them to the grid operation, the techniques based on mathematical programming such as power flow and optimal power flow (OPF) are more appropriate than exhaustive search and heuristics methods [3].

Though the OPF algorithms are efficient at analyzing the active distribution planning, they require adopting heuristic techniques to solve high-dimensional non-convex problem or linear convex relaxation of power flow equations. A two-step heuristic algorithm with master and slave problem has been studied in [4]. This study adopts a heuristic algorithm to solve the optimal siting and sizing problem of ESSs and then uses an AC OPF technique to consider optimal voltage control while reducing the total energy and network losses. In [5], a genetic algorithm (NSGA-II) has been used to identify the optimal place, size, and scheduling of energy storage in the distribution network. In the paper, a full MO (Multi-Objective) optimization procedure has been developed to identify the Pareto set of design options with fixed network topology for a given MV network. Another heuristic approach has been found in [6], where it has been analyzed a grid-connected storage for an MO problem considering both distribution and transmission network objectives. However, heuristic techniques often required high computational burden and are not guaranteed to converge in global optima [7]. Convex relaxation techniques have been developed to obtain an acceptable solution while ensuring algorithmic efficiency. The two most commonly used relaxations for distribution network are Semi-definite Program (SDP) and Second Order Cone Programming (SOCP). Though both SOCP and SDP have been proven to be exact under certain conditions [8]-[9], SOCP has been considered in this paper due to its higher algorithmic performances.

In this paper, RO optimization has been integrated with ESS optimal location in order to find a set of solutions that are robust against the variations of uncertain quantities. The final goal is to evaluate how ESS can help in operating of the network even in uncertain scenarios. The proposed methodology for ESS investment in the distribution network is based on a multi-period AC OPF algorithm.

The comparison between the deterministic formulation and the RO of such OPF has been made by using the SOCP convex relaxation to make it faster without losing in effectiveness. This relaxation involves equality constraints and replacing certain quadratic terms with linear terms as detailed in the next sections of the paper.

The methodology has been tested on a distribution network that is representative of a class of distribution networks and derived from the ATLANTIDE project [10].

DETERMINISTIC FORMULATION OF ENERGY STORAGE PLANNING

The proposed methodology for storage investment in the distribution network is based on a multi-period AC OPF algorithm. The objective function (OF) consists of minimizing the operational extra-cost that should be sustained for complying with the technical constraints. Such cost includes the penalty terms for RES (C_n^{RESc}) and for biomass combined heat and power (CHP) curtailment (C_n^{CHPc}), and the cost for the demand response (C_n^{DR}). Furthermore, since the goal of the paper is to evaluate the contribution of energy storages to the management of the network, even in uncertain conditions, the investment cost $C_n^{CAPEX\_ESS}$ to be sustained for the storages allocated in the network is added to the operational cost, as in (1).

$$\min C_{tot} = \min\{\sum_{n=1}^{N}[C_n^{RESc} + C_n^{CHPc} + C_n^{DR} + C_n^{CAPEX\_ESS}]\} \quad (1)$$

Subject to voltage and current limits, power flow equations, and storage technical constraints.

In the following each cost term and constraints are detailed.

Penalty for RES curtailment $\mathbf{C_n^{RESc}}$

The cost of curtailed energy from RES due to network constraint violations has been monetized as twice the price of energy paid in the wholesale market c_{EN} (here, 58 €/MWh), as in (2).

$$C_n^{RESc} = \sum_{t=1}^{T} 2 \cdot c_{EN} \cdot P_n^{RESc}(t) \qquad n = 1 \cdots N \quad (2)$$

where $P_n^{RESc}(t)$ is the energy curtailed at the time interval t by the RES generator connected to the n-th bus of the network.

Since the increment of network hosting capacity may be quantified via the possibly avoided curtailment of RES production, the smaller this term, the better the storage allocation solution.

Penalty for biomass CHP curtailment $\mathbf{C_n^{CHPc}}$

This cost is assumed as the fuel cost F [€/MWh] (here, 80 €/MWh) increased by 20%, as in (3).

$$C_n^{CHPc} = \sum_{t=1}^{T} 1.2 \cdot F \cdot P_n^{CHPc}(t) \qquad n = 1 \cdots N \quad (3)$$

where $P_n^{CHPc}(t)$ is the energy curtailed at the time interval t by the biomass CHP connected to the n-th bus of the network.

Demand response cost $\mathbf{C_n^{DR}}$

Regarding the term referred to the Customers, in this paper only the cost of shaving the peak loads has been considered, by assuming that it is not possible to fully control the customer demand but only cut a quote of their consumption in some critical conditions. It is assumed, as the RES curtailment, that this curtailed energy is paid at twice the energy price c_{EN}, as in (4).

$$C_n^{DR} = \sum_{t=1}^{T} 2 \cdot c_{EN} \cdot P_n^{DR}(t) \qquad n = 1 \cdots N \quad (4)$$

where $P_n^{DR}(t)$ is the energy curtailed at the time interval t to the customer connected to the n-th bus of the network.

Storage investment cost $\mathbf{C_n^{CAPEX\_ESS}}$

The storage investment cost (SC_n) is a function of the size of the storage in terms of rated power and energy as in (5).

$$SC_n = c_P \cdot P_n^{rated} + c_E \cdot E_n^{rated} \qquad n = 1 \cdots N \quad (5)$$

where c_P and c_E are the specific costs of the ESS adopted technology, reliant respectively on the power rating P_n^{rated} and the nominal capacity E_n^{rated} of the n-th ESS located in the network (here c_P =200 €/kW and c_E =400 €/kWh according to the market cost of Lithium-ion technology).

In order to consider this cost in the objective function (1) only a daily quote of SC_n is added to the operational terms of (1), calculated as in (6).

$$C_n^{CAPEX\_ESS} = \frac{K_S}{365} \cdot SC_n \qquad n = 1 \cdots N \quad (6)$$

where K_s is a capital recovery factor (here K_s=0.1, for considering 10 years as ESS lifetime).

In this paper, it is assumed that the storages are DSO owned and managed for relieving contingencies. Thus, the ESS OPEX (operational expenditures) is not considered in the optimization. According to this point of view, it is supposed that the minimization of operational cost represents the only incomes that allow DSO to pay back EES CAPEX (capital expenditures) and operational expenditures. Actually, a term that takes into account the depreciation of the ESSs due to their use could be included, but in this paper this cost is assumed negligible.

Minimization constraints and relaxation

In the proposed multi-temporal AC OPF model, the SOCP convex relaxation has been used. The active and reactive power flows in the proposed OPF problem is formulated as in eq. (7) and (8).

$$P_n^g(t) + P_n^{RES}(t) - P_n^{RESc}(t) + P_n^{CHP}(t) - P_n^{CHPc}(t) - PD_n(t) + P_n^{DR}(t) - P_n^c(t) + P_n^d(t) - \sum_{m\epsilon\theta_n} R_{mn} \cdot I_{mn}^2 = \sum_{m\epsilon\theta_n} P_{mn}(t) \quad (7)$$

$$Q_n^g(t) + Q_n^{RES}(t) - Q_n^{RESc}(t) + Q_n^{CHP}(t) - Q_n^{CHPc}(t) - QD_n(t) - \sum_{m\epsilon\theta_n} X_{mn} \cdot I_{mn}^2 = \sum_{m\epsilon\theta_n} Q_{mn} \quad (8)$$

Where $(P_n^{RES}(t); Q_n^{RES}(t))$ and $(P_n^{CHP}(t); Q_n^{CHP}(t))$ define the expected RES and CHP production, $PD_n(t)$ and $QD_n(t)$ are the active and reactive power delivered to the load connected to the n-th node, $I_{mn}(t)$, $P_{mn}(t)$, and $Q_{mn}(t)$ are respectively the current, the active and the reactive power flowing from m-th bus to the n-th one, $Q_n^{RES}(t)$ is the reactive power provided by PV and wind, and R_{mn} and X_{mn} are the resistance and reactance of the mn-th branch. $P_n^g(t)$ and $Q_n^g(t)$ are the active and reactive power provided by the upstream connections (first node of the network). The values of $P_n^g(t)$ and $Q_n^g(t)$ are zero except for the first node. The current magnitude quadratic term can be defined as function of the corresponding active and reactive power quadratic terms (eq. (9)-(11)).

$$I_{mn}^2 \geq \frac{P_{mn}^2 + Q_{mn}^2}{V_n^2} \quad (9)$$

$$P_{mn}^2(t) + Q_{mn}^2(t) = S_l^2(t) \quad (10)$$

$$i_{mn}(t) \cdot v_{mn}(t) = S_l^2(t) \quad (11)$$

The equality constraints in this model, eq. (9), are relaxed ultimately by relaxing the magnitude of currents within each branch and using a conic formation on the limitation of exchanged active power.

For linearization purposes, the quadratic terms of voltage and current magnitude have been replaced with the linear ones as in (12).

$$I_{mn}^2 = i_{mn}\ ; \qquad V_{mn}^2 = v_{mn} \quad (12)$$

The new variables (i_{mn}, v_{mn}) successfully formulate the SOCP problem cording with the following constraints.

$$V_{min}^2 \leq v_n(t) \leq V_{max}^2 \quad (13)$$

$$P_n^{min\ RESc/CHPc} \leq P_n^{RESc/CHPc}(t) \leq P_n^{maxRESc/CHPc} \quad (14)$$

$$Q_n^{min\ RESc/CHPc} \leq Q_n^{RESc/CHPc}(t) \leq Q_n^{maxRESc/CHPc} \quad (15)$$

The eq. (13) provides the voltage limits of each bus. The eq. (13)-(15) limit the active and reactive power curtailment associated with RES and CHP generators. Furthermore, the constraints about storages may be formulated as in (16)-(20).

$$SOC_n(t) = SOC_n(t-1) + \left(P_n^c(t) \cdot \eta_c - \frac{P_n^d(t)}{\eta_d}\right) \cdot \Delta t \quad (16)$$

$$0 \leq P_n^c(t) \leq \alpha_n^c \cdot P_n^{c,max}(t) \quad (17)$$

$$0 \leq P_n^d(t) \leq \alpha_n^d \cdot P_n^{d,max}(t) \quad (18)$$

$$SOC_{n,min} \leq SOC_n(t) \leq SOC_{n,max} \quad (19)$$

$$\alpha_n^c(t) + \alpha_n^d(t) \leq 1 \quad (20)$$

where $\alpha_n^c(t)$ ϵ [0 or 1] and $\alpha_n^d(t)$ ϵ [0 or 1].

The state of charge (SOC) of energy storage devices is calculated by considering the initial state of charge and charging and discharging efficiencies η_c and η_d (eq. (16)). To restrict the maximum charging and the depth of discharging and for avoiding the simultaneous charging and discharging, the binary variables α_n^c and α_n^d, of which only one can be different from zero, have been considered in eq. (17)-(20). An additional constraint is added to force the SOC of the first time-step to be the same to the SOC of the last one of the time horizon T as represented by eq. (21).

$$SOC_{n,0} = SOC_{n,T} \quad (21)$$

It is worth mentioning that during the estimation of charging and discharging power of the storage unit, a quadratic term has arisen due to the multiplication of binary and integer variables. A decomposition technique has been used to linearize the relevant constraints by rewriting constraints in the form of (22) as in (23) and (24) to avoid the bilinear terms.

$$x <= y * z * c \quad (22)$$

$$x <= y * z_{max} * c_{max} \quad (23)$$

$$x <= z * c \quad (24)$$

for x continuous, y binary, z integer variable in $[0, x_{max}]$ and $[0, z_{max}]$.

ROBUST OPTIMIZATION FORMULATION

The robust model takes into account the set of values for the uncertain parameters, named *uncertainty set.* The selection of uncertainty solely depends on the available information of uncertain parameters and the level of robustness acceptable by the decision maker. A compromise between the robustness against each physical realization of the uncertain parameters and the size of the uncertainty set should be reached for making reasonable planning problems. The most robust uncertainty set which guarantees that the constraints are never violated is the *box uncertainty set*. However, this kind of sets only considers the worst scenarios that often make the model very conservative, leading to unacceptable solutions [11]. Since box uncertainty set is often too pessimistic, two other uncertainty sets are used in practice: the *ellipsoidal* and the *polyhedral uncertainty set*. The ellipsoidal uncertainty set leads to a better objective value for a fixed probability guarantee. However, it could lead to a quadratic constraint from a linear problem. In this paper, the *polyhedral uncertainty set* which considers being tractable from a computational point of view is used. This set can be expressed as in (25).

$$U = \{\xi \in \mathbb{R}^L : \|\xi_\infty\| \leq \Gamma\} \quad (25)$$

where the real vector of dimension L ξ is the only knowledge available, namely *perturbation vector*, that varies inside a given interval. $\|\xi_\infty\|$ defines the continuous uniform norm of ξ and Γ is the measure of the uncertainty. This type of uncertainty set also called as a *budgeted uncertainty set* since the level of robustness can be adjusted with Γ. It is important to properly select the budget of uncertainty Γ in order to have a reasonable solution maintaining sufficient robustness of the model. In this work, the polyhedral uncertainty set has been adopted since it produces sufficiently robust solution if the budget of uncertainty is chosen based on the uncertainty level one wants to accept. As Γ increases, more uncertain the considered scenario, and less risky becomes the solution.

RO problem usually contains an infinite number of constraints due to imposing worst case formulation and hard constraints. Therefore, it is often computationally unfeasible. Generally, there are two ways to deal with this kind of situation [12]. One way to deal with this issue is using robust reformulation techniques to make the formulation immune of all the uncertain parameters, adopted in this paper. Since the uncertainty is constraint-wise, the reformulation will only deal with constraints, which contain the uncertain parameters. The robust model structure and its OF is identical as in the deterministic model. The changes will arise in the constraints involving the uncertainty. In this paper, three uncertain parameters are considered, the forecasted real power of wind farms P_w, the expected real power of PV plant P_{pv} and demand PD_n. The uncertain parameter, indicated with tilde (~), are $\widetilde{P_{w,t}}$, $P_{pv,t}$ and $\widetilde{P_{D,t}}$ that represent the uncertain wind power output, the uncertain PV output and the uncertain load demand at each time step t respectively. ΔP_w, ΔP_{pv} and ΔP_D are their maximum deviations from the expected values. The polyhedral uncertainty set for wind power output can be defined as in (26).

$$U^w = \{\widetilde{P_{w,t}}: \xi_{w,t} \in \mathbb{R}^L \; s.t \|\xi_\infty\| \leq \Gamma_w\} \quad \widetilde{P_{w,t}} \in [P_{w,t} + \Delta P_{w,t} \cdot \xi_{w,t}] \tag{26}$$

where ξ_w is the degree of uncertainty of the wind power output. In other words, it can be considered as the quantification of the actual deviation from the forecasted value P_w and it belongs to the interval$[-1; 1]$; Γ_w is the budget of wind generation uncertainty that lies between 0 to 1, where 0 being the deterministic case and 1 defined the most robust case. Due to the space limitation, the reformulation process of PV has been omitted but the process is completely analogous to wind power. The similar formulation will be applied to load demand uncertainty as in (27).

$$U^D = \{\widetilde{P_{D,t}}: \xi_{D,t} \in \mathbb{R}^L \; s.t \|\xi_\infty\| \leq \Gamma_D\} \quad \widetilde{P_{D,t}} \in [P_{D,t} + \Delta P_{D,t}\xi_{D,t}] \tag{27}$$

As for the case of wind, the larger Γ_D, the larger the uncertainty set, and the larger the worst-case value of the uncertain component of the constraint.

The reformulation process is composed of three main steps: (i) worst case reformulation, (ii) duality and (iii) robust counterpart.

Worst case reformulation

First, it is important to identify the maximum deviation from the nominal value and rewrite the constraints that are affected by the uncertain parameters such as a way that it considers the possible worst case. In the worst case of maximum load and minimum production, the value of ξ_D will be 1 and the values of ξ_w and ξ_{pv} will be -1. Thus, the worst case has been formulated as inner problems of maximization of the demand deviation and minimization of the power production deviation. In Table I the inner problems and their new formulation for demand, wind and PV, respectively, are reported. Additional variables $M_{D,t}(M_{D,t} \geq 0)$, $M_{w,t}(M_{w,t} \leq 0)$ and $M_{pv,t}(M_{pv,t} \leq 0)$ are introduced to relax the absolute term.

Table 1: Worst case formulation of uncertain parameters

Demand:	$\max(\Delta P_{D,t} \cdot \xi_{D,t})$ $s.t. \lvert\xi_{D,t}\rvert \leq \Gamma_D$ $0 \leq \lvert\xi_{D,t}\rvert \leq 1$	$\max(\Delta P_{D,t} \cdot \xi_{D,t})$ $s.t. \mathrm{M}_{D,t} \leq \Gamma_D$ $\mathrm{M}_{D,t} \geq \xi_{D,t}$ $-1 \leq \xi_{D,t} \leq 1$
Wind:	$\min(\Delta P_{w,t} \cdot \xi_{w,t})$ $s.t. \lvert\xi_{w,t}\rvert \geq \Gamma_w$ $\lvert\xi_{w,t}\rvert \leq 1 \quad \forall w$	$\min(\Delta P_{w,t} \cdot \xi_{w,t})$ $s.t. \mathrm{M}_{w,t} \geq -\Gamma_w$ $\mathrm{M}_{w,t} \leq \xi_{w,t} \quad \forall w$ $-1 \leq \xi_{w,t} \leq 1 \quad \forall w$
PV:	$\min(\Delta P_{pv,t} \cdot \xi_{pv,t})$ $s.t. \lvert\xi_{pv,t}\rvert \geq \Gamma_{pv}$ $\lvert\xi_{pv,t}\rvert \leq 1 \quad \forall pv$	$\min(\Delta P_{pv,t} \cdot \xi_{pv,t})$ $s.t. \mathrm{M}_{pv,t} \geq -\Gamma_w$ $\mathrm{M}_{pv,t} \leq \xi_{pv,t} \quad \forall pv$ $-1 \leq \xi_{pv,t} \leq 1 \quad \forall pv$

Moreover, each uncertain parameter will be solved separately.

Forming Dual

In order to make the problem computationally tractable, this step aims at finding the dual of the inner minimization/maximization problems. Their duals, even the dual variables do not have any physical meaning, yield the same optimal objective value by strong duality theorem. Therefore, the inner optimizations can be reformulated as in Table 2, where Q, G, I and R are the dual positive variables respectively associated with the primal optimization problem (the subscripts are related to the demand, wind and, PV respectively).

Table 2: Dual forming of uncertain parameters

Demand:	$min\{Q_{D,t} \cdot \Gamma_D + G_{D,t} + I_{D,t}\}$ $s.t \; R_{D,t} + G_{D,t} - I_{D,t} = \Delta P_{D,t}$ $Q_{D,t} - R_{D,t} \geq 0$
Wind:	$max\{-Q_{w,t} \cdot \Gamma_w - G_{w,t} - I_{w,t}\}$ $s.t. R_{w,t} - G_{w,t} + I_{w,t} = \Delta P_{w,t} \quad \forall w$ $Q_{w,t} - R_{w,t} \geq 0 \quad \forall w$
PV:	$max\{-Q_{pv,t} \cdot \Gamma_{pv} - G_{pv,t} - I_{pv,t}\}$ $s.t. R_{pv,t} - G_{pv,t} + I_{pv,t} = \Delta P_{pv,t} \quad \forall pv$ $Q_{pv,t} - R_{pv,t} \geq 0 \quad \forall pv$

Forming the robust counterpart

The inner optimization problems for demand, wind, and PV are ready to be integrated into the main deterministic model. To get the robust counterpart of the original deterministic model, the objective functions of the dual form have to be added in the respective constraint and the constraints of the dual form will need to be included in the algorithm. All the constraint containing uncertain parameter can be replaced with linear constraints without uncertainty and converted into a mixed integer form. Since in the main deterministic model, only one constraint is affected by uncertainty, namely the power balance equation (7), the reformulated power balance equation becomes as in (28).

$$P_n^{CHP}(t) - P_n^{CHPc}(t) - P_n^{RESc}(t) + P_n^{DR}(t) + P_n^d(t) - P_n^c(t) - PD_n(t) - Q_{D,t} \cdot \Gamma_D + G_{D,t} + I_{D,t} + P_w(t) - Q_{w,t} \cdot \Gamma_w - G_{w,t} - I_{w,t} - R_{mn} \cdot I_{mn}^2 = \sum_{m \epsilon \theta_n} P_{mn}(t) \tag{28}$$

The additional constraints for load demand, wind, and PV that will be added in the algorithm are reported in (29)-(34).

Demand:

$$R_{D,t} + G_{D,t} - I_{D,t} = \Delta P_{D,t} \tag{29}$$

$$Q_{D,t} - R_{D,t} \geq 0 \tag{30}$$

Wind:

$$R_{w,t} - G_{w,t} + I_{w,t} = \Delta P_{w,t} \quad \forall w \tag{31}$$

$$Q_{w,t} - R_{w,t} \geq 0 \quad \forall w \tag{32}$$

PV:

$$R_{pv,t} - G_{pv,t} + I_{pv,t} = \Delta P_{pv,t} \quad \forall pv \tag{33}$$

$$Q_{pv,t} - R_{pv,t} \geq 0 \quad \forall pv \tag{34}$$

The new model does not contain any uncertainty and is formulated as a mixed integer second order conic programming (MISOCP) problem that can be solved efficiently using CPLEX that uses a branch and cut algorithm to find the integer feasible solution.

CASE STUDY

The procedure has been applied to a test distribution network derived from the ATLANTIDE project [10]. The MV network, representative of the industrial ambit, is constituted by 100 nodes, subdivided in 7 feeders. The total demand is about 30 MVA (372 GWh/year) and the total installed DG capacity is 34 MW (27.2 GWh/year), as mix of wind, PV and biomass CHP generators.
The generation and load profiles were simulated according to the ATLANTIDE annual profiles. Furthermore, in this paper, the annual profiles have been reduced to twelve typical day profiles, differentiated between working days, Saturdays and holidays (Sundays included), and between seasons, for each kind of customers (i.e., industrial, residential, commercial and agricultural) and for each technology of DG (i.e., wind turbine, PV and CHP).

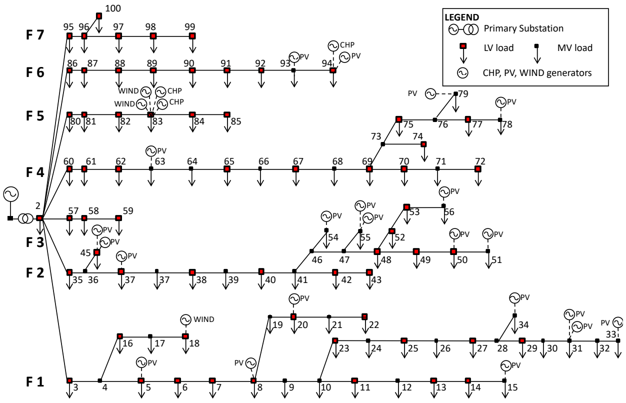

Fig. 1: Test network

The mathematical formulation of the RO for an AC OPF based energy storage planning tool has been programmed in GAMS and solved using CPLEX on a 2.30 GHz personal computer with 4GB RAM. In this experimental study, the worst case has been considered when the load is high ($\xi_{D,t}$=1) and wind and PV generation is low ($\xi_{w,t}$=-1).
For the sake of brevity, in the following, the results obtained by the application of the described optimization to one typical day, the winter working days, have been reported. The time horizon of 24h has been considered with a time step of 1h. Three scenarios have been considered: the certain one (solved by the deterministic OPF) and two uncertain scenarios with different values of risk ($\Gamma = 0.5$ and $\Gamma = 1$), both solved with RO.
In this typical day some under voltage conditions occur and, thus, for solving these issues it is necessary to resort to the load peak shaving. ESSs prove to be useful for reducing the curtailment of the demand.

Storage Allocation

All the buses of the test network were assumed candidates for storage placement. The available ESS were considered of 1.0 MW/2h storage capacity. The efficiencies for charging and discharging were considered 90% each, which gives an overall roundtrip efficiency of 81%.
Table 3 shown which busses are chosen by the algorithms for the ESS installation.

Table 3: ESS location in the test network of Fig. 1

Scenarios	ESS Location
Certain scenario $\mathbf{\Gamma = 0}$	14
Medium risk scenario $\mathbf{\Gamma = 0.5}$	14, 67
No risk scenario $\mathbf{\Gamma = 1}$	11, 14, 67

In the deterministic model, any consideration of uncertainty is avoided by assuming perfect information for all parameters. Such a certain case ($\Gamma = 0$) suggests 1 storage device to be installed in the network. As the budget of uncertainty increases, the number of storages increases from 2 ($\Gamma = 0.5$) to 3 in the no risk scenario that includes the worst case. By comparing the three scenarios, it is worth noticing that the results are substantially incremental: the medium risk scenario includes the location of the certain one and the robust scenario (no risk) includes in turn the intermediate one. In the robust scenario (no risk), it is observed that the ESS locations in the feeder F1 of Fig. 1 are two instead of the only one installed in the more certain cases.
Table 4 summarizes the daily operational costs for the four examined scenarios and the quantity of shed demand for solving the remaining contingencies. The peak shaving drastically decreases by using the ESS of the certain scenario (deterministic OPF) and then it is progressively further reduced in the uncertain scenarios. A significant reduction of daily operational cost has been observed with the ESS inclusion in the deterministic OPF and in the medium risk scenario. The resort to load shedding is so much reduced (-50% in the certain scenario and even -85% in the intermediate scenario) that the CAPEX for ESS installation does not negatively impact in the final cost. Hence, operational cost, even in the medium risk solutions, is reduced since the penalty for load shedding is even higher the investment cost of storages. The no risk scenario requires one more ESS to be installed in the network, thus, the daily operational cost increases. This demonstrates that the ESS in this system help to avoid the peak shaving of loads but for relieving even the worst case and reducing even more the resort to load shedding it becomes necessary an expensive investment.

Table 4: Daily operation cost of the test network

Scenarios	Operational cost [€/day]	Load shedding [MWh/day]
No storage scenario	811.61	6.99
Certain scenario $\mathbf{\Gamma = 0}$	683.89	3.53
Medium risk scenario $\mathbf{\Gamma = 0.5}$	667.57	1.03
No risk scenario $\mathbf{\Gamma = 1}$	834.35	0.12

CONCLUSIONS

In this study, an approach of applying robust optimization on an AC OPF based siting and sizing of energy storage devices in the distribution networks has been proposed.

Since DC OPF neglects the losses and may lead to an infeasible planning solution, the use of AC OPF in this study aimed at increasing accuracy in the planning.
The polyhedral uncertainty set has been exploited for representing the uncertainty sets due to its flexibility to find a trade-off between economic efficiency and conservatism. The integration of this kind of flexibility also helped to observe scenarios other than the worst-case one that most of the robust optimization problems do not consider.
By considering the worst-case scenario only, such problems do not provide an optimal solution rather a very conservative solution which could be unrealistic. However, the analytical reformulation technique helped to find the robust counterpart of the original problem that was solved with less computational burden using CPLEX.
Since planning involves limited economic budget and resources, this study will provide a comprehensive view which is a combination of different scenario (budget of uncertainty). Moreover, the decision maker could also consider the external factors that are not considered in the model such as land use for storage placement, in the decision-making process to get the optimal solution by selecting the budget of uncertainty based on the planner's perspective.

Acknowledgments

Nayeem Chowdhury has been funded from the European Union's Horizon 2020 research and innovation programme under Grant Agreement No 676042.
The contribution of G. Pisano to this paper has been conducted within the R&D project "Cagliari2020" partially funded by the Italian University and Research Ministry (grant# MIUR_ PON04a2 _00381).

REFERENCES

[1] Hartmann, B., & Dan, A., 2014, "Methodologies for storage size determination for the integration of wind power", *IEEE Trans. on Sustainable Energy*, vol. 5(1), 182-189.
[2] Jabr, R. A., Džafić, I., & Pal, B. C., 2015, "Robust optimization of storage investment on transmission networks", *IEEE Trans. on Power Systems*, vol. 30(1), 531-539.
[3] Grover-Silva, E., Girard, R., & Kariniotakis, G., 2018, "Optimal sizing and placement of distribution grid connected battery systems through an SOCP optimal power flow algorithm", *Applied Energy*, vol. 219, 385-393.
[4] Nick, M., Hohmann, M., Cherkaoui, R., & Paolone, M., 2013, "Optimal location and sizing of distributed storage systems in active distribution networks", *Proc. IEEE PowerTech conference,* Grenoble (UK).
[5] Celli, G., Pilo, F., Pisano, G., & Soma, G. G., 2018, "Distribution energy storage investment prioritization with a real coded multi-objective Genetic Algorithm", *Electric Power Systems Research*, vol. 163, 154-163.
[6] Motalleb, M., Reihani, E., & Ghorbani, R., 2016, "Optimal placement and sizing of the storage supporting transmission and distribution networks", *Renewable Energy*, vol. 94, 651-659.
[7] Paudyal, S., Canizares, C. A., & Bhattacharya, K., 2011, "Three-phase distribution OPF in smart grids: Optimality versus computational burden", *Proc. IEEE PES ISGT Europe, 2011*
[8] Bose, S., Low, S. H., Teeraratkul, T., & Hassibi, B., 2015, "Equivalent relaxations of optimal power flow", *IEEE Trans. on Automatic Control*, vol. 60(3), 729-742.
[9] Gan, L., Li, N., Topcu, U., & Low, S. H., 2015, "Exact convex relaxation of optimal power flow in radial networks", *IEEE Trans. on Automatic Control*, vol. *60*(1), 72-87.
[10] Bracale, A., Caldon, R., Celli, et Al., 2012, "Analysis of the Italian distribution system evolution through reference networks", *Proc. IEEE PES ISGT Europe, 2012*.
[11] Ben-Tal, A., El Ghaoui, L., & Nemirovski, A., 2009, *Robust optimization* (Vol. 28). Princeton University Press.
[12] Gorissen, B. L., Yanıkoğlu, İ., & den Hertog, D., 2015, "A practical guide to robust optimization", *Omega*, vol. 53, 124-137.